Q8: 15.6 and 7

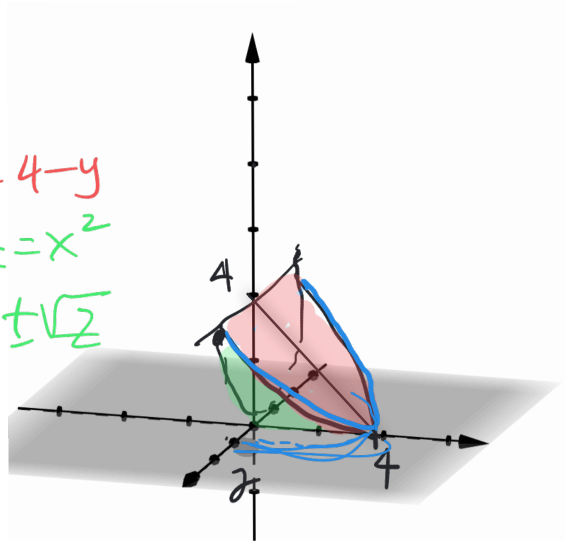
(1) SET UP BUT DO NOT EVALUATE: integrals as specified to find the volume of the solid bounded by the cylinder $z = x^2$ and the planes y=0, and z=4-y.

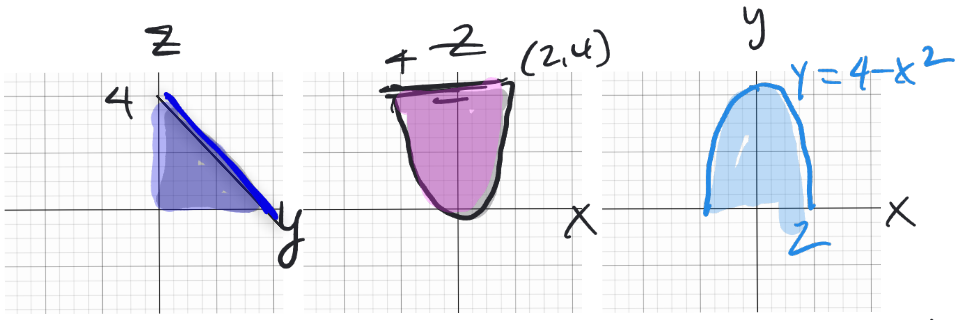
a) sketch the solid and sketch the projections in each of the coordinate planes (Note: This is very similar to the example done on video) (12 POINTS)

Top $z=4-y$

Bottom $z=x^2$

$x=\pm\sqrt{z}$




For projection in xy plane, eliminate z

$$\begin{cases} z=4-y \\ z=x^2 \end{cases} \quad x^2=4-y \quad y=4-x^2$$

b) Triple integral- rectangular coordinates; order dz dy dx

$$\int_{-2}^{2}\int_{0}^{4-x^2}\int_{x^2}^{4-y} dz\,dy\,dx$$

c) Triple integral- rectangular coordinates; order dx dy dz

$$\int_{0}^{4}\int_{0}^{4-z}\int_{-\sqrt{z}}^{\sqrt{z}} dx\,dy\,dz$$

(d ) Triple integral- rectangular coordinates; order dy dz dx

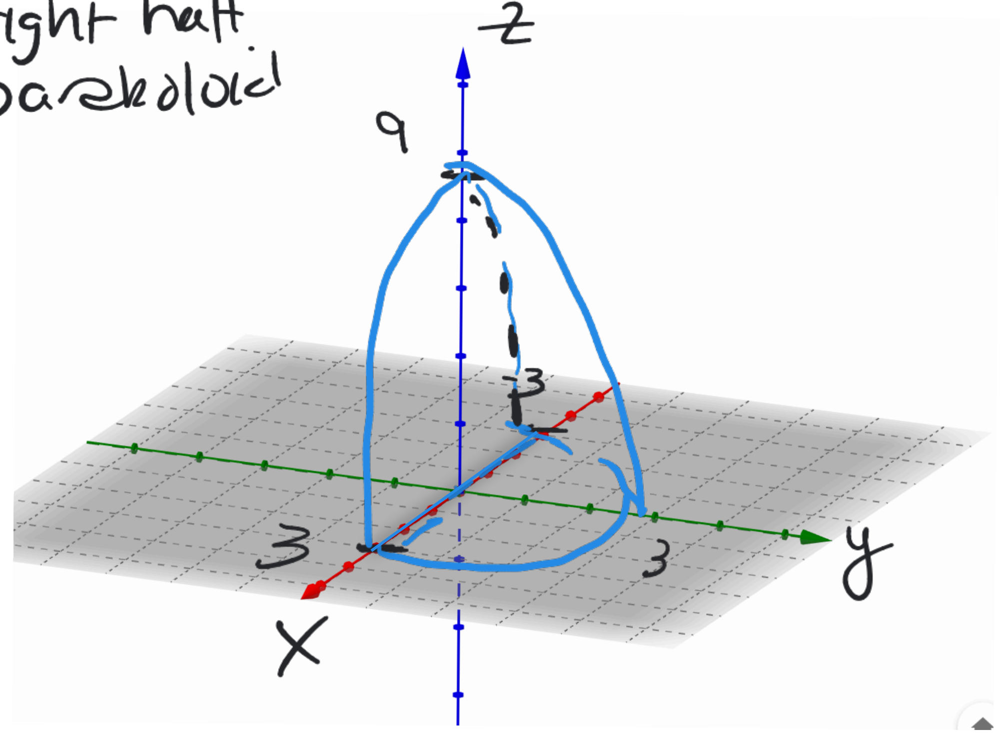
$$\int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{4-z} dy\,dz\,dx$$

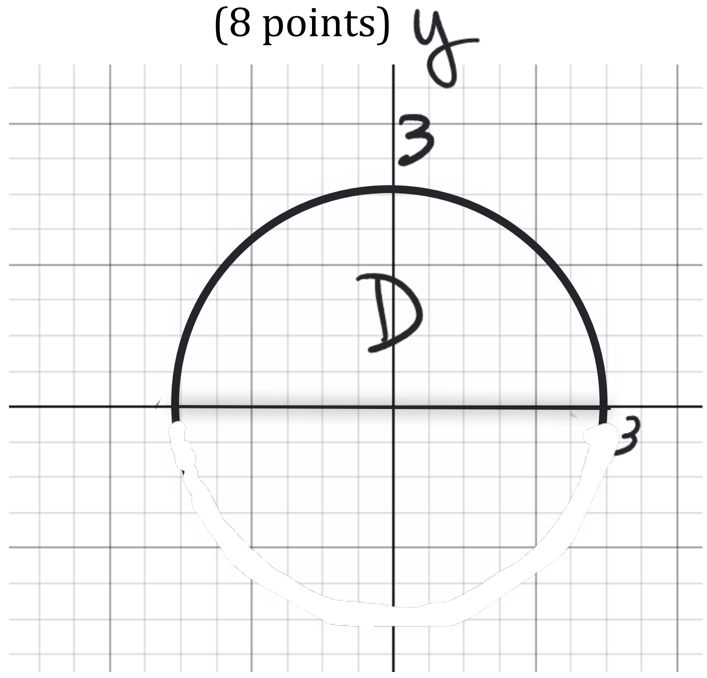
$r \quad dV = r \; dz \, r \, dr \, d\theta$

$= r^2 \, dz \, dr \, d\theta$

(2) Evaluate the integral $\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2+y^2} \, dz \, dy \, dx$ by switching to cylindrical coordinates (8 points)

right half paraboloid




$$\int_0^{\pi} \int_0^{3} \int_0^{9-r^2} r^2 \, dz \, dr \, d\theta$$

$$\int_0^{\pi} \int_0^{3} r^2 \; z \Big]_0^{9-r^2} dr \, d\theta$$

$$= \int_0^{\pi} \int_0^{3} r^2 (9-r^2) \, dr \, d\theta = \int_0^{\pi} \int_0^{3} (9r^2 - r^4) \, dr \, d\theta$$

$$= \int_0^{\pi} 3r^3 - \frac{1}{5} r^5 \Big]_0^{3} d\theta = \int_0^{\pi} 3^4 - \frac{3^5}{5} \, d\theta = \int_0^{\pi} \frac{162}{5} \, d\theta = \frac{162\pi}{5}$$